1 Breaking bar

A linearly imposed compression wave of amplitude $-4\sigma_0$ is traveling through a bar of elastic modulus E and density ρ. The wave is reflected at the free boundary on the right-hand side (see Figure 1). Determine the average coordinate x_r where fracture will occur along the bar knowing that the tensile yield strength is $2\sigma_0$.

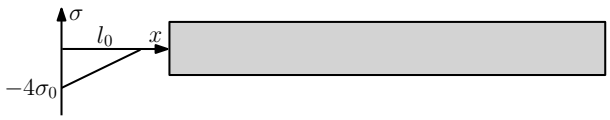

Figure 1: Linearly increasing compression wave in a bar.

2 Energy conservation

A sinusoidal compressive wave is applied at $t = 0$ on the left boundary of a bar of length L and is described by: $\sigma = \sigma_0 \cdot \sin\left(2\pi \frac{ct}{L}\right)$ (see figure below). The right-hand side boundary is free. Express the general expression of the energy in this system.

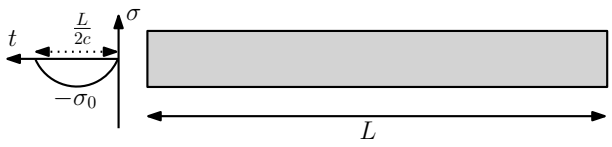

Figure 2: Sinusoidal compressive wave in a bar.

3 Chain of atoms

Let's consider the one dimensional atomic chain presented in Figure 3. The characteristics of this system are the stiffness K, the inter-atomic distance a, and the atom mass M_1, M_2.

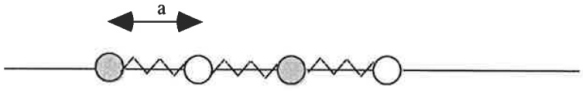

Figure 3: Mass-spring system.

Question 1

Suggest a waveform for these two types of atoms. Find the dispersion relation for this heterogeneous atomic chain, in the system direction.

Question 2

For the case $ka << 1$, find a simplified form of the dispersion equation. What would be the wave speed in an equivalent homogeneous media? What happens when ka tends toward zero?